Monte Carlo Simulations for Tilted-Channel Electron Multipliers
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Abstract—Microchannel electron multipliers with tilted structures are simulated using the Monte Carlo method. Gains of secondary electrons are calculated for different structures of the electron multiplier. For a short tilted cylindrical channel of the electron multiplier, a maximum gain is achieved greater than $10^6$ at a tilt angle near $25^\circ$. The maximum gain is about $10^5$ times larger than that of the nontilted channel. An explanation for the improvement of gain in tilted channel is suggested.

Index Terms—Electric fields, electron multiplier, Monte Carlo methods, secondary electron emission, simulation.

I. INTRODUCTION

MICROCHANNEL electron multipliers [1]–[3] are applied to various areas, such as detectors, scanning tunneling microscopes, image intensifiers, and displays. Electron multipliers amplify the input current of electrons through complicated stochastic processes. When an electron enters a channel of an electron multiplier to which different voltages are applied at both ends, it strikes the inside wall of the channel with some collision energy and a few secondary electrons are emitted from the channel wall. These emitted secondary electrons are accelerated along the voltage gradient. Each secondary electron hits the wall with a collision energy obtained while moving along the voltage gradient and produces new secondary electrons. The process is repeated until all secondary electrons escape the channel. The repeated steps of the above process result in a cascade process and in multiplication of secondary electrons.

High gain of electron multipliers, which is defined as the multiple of output current for the input current, is required in practical applications. The high gain can be obtained by 1) using a material with high secondary electron yield, 2) increasing an aspect ratio (=length/diameter of channels), or 3) applying a high voltage. First, Mearini et al. [4], [5] suggested diamondlike carbon as a high-yield material. They reported that the yield was greater than 50 while yields for typical materials are less than ten. Second, electron multipliers with high aspect ratio, 40 to 60, can produce high gains [3]. The multipliers with long channels, however, require great efforts in manufacturing process [3]. Third, high applied voltages also improve the gain, though simulation results [6] show that there is a particular voltage at which the gain has a peak. Additionally, the use of high voltages is restricted because high voltage may start ion feedback that causes noise in output signal due to outgases in channels [3], [7].

Here, we numerically investigate a method to improve the gain of an electron multiplier by changing the geometry of channels. In a typical electron multiplier the electric field due to the applied voltage is parallel with the axis of the channels. In our geometry, however, the channels are tilted to the electric field. In tilted channels, there is a small normal component of electric field to the surface of the channel wall. This weak electric field has an effect on electron trajectories and improves the gain of electron multipliers.

In Section II we describe a computational model of electron multipliers using the Monte Carlo (MC) method [8]. In Section III we discuss numerical results through MC simulations. Finally we finish writing with conclusions in Section IV.

II. DESCRIPTIONS OF THE MODEL

A tilted microchannel electron multiplier consists of an array of channels, shown in Fig. 1, where the channels are tilted with angle $\theta$, $V_0$ is the applied voltage, $L$ is the length between faces of multiplier plate, and $D$ is the diameter of a channel. Typically, a few thousand volts are applied between the faces of multiplier. The voltage difference supplies the necessary energy to electrons for the release of secondary electrons. The voltage difference plays another role in electron multipliers. The power supply maintaining this voltage difference provides electrons to the electron multiplier to replace the emitted secondary electrons. In a typical multiplier the electric field due to the voltage difference is approximately parallel to the axis of the channels.

Here we consider the simulation [6], [7], [9], [10] of electron multiplication in a single channel within an electron multiplier. We approximate that the electric field is uniform by neglecting fringe fields near the input and output end of the channel. Although the fringe fields will affect the electron trajectories, we do not believe that the fields in our geometry will significantly change our results. We will not consider the space charge effect in our simulations, so that the cases of more than $10^6$ gain are beyond the simulation for our typical channel of diameter of 100 $\mu$m and applied field of 2 $V/\mu$m. According to Loty [11], the maximum gain may be estimated $10^6$ using the formula of maximum charge, $Q_{\text{max}} = \pi \varepsilon_0 r^2 E$, where $\varepsilon_0$ is the dielectric constant of the air, $r$ is the radius of channel, and $E$ is the field strength near the output end. To get the gain of the multiplier, we need to calculate trajectories, kinetic energies, and incident angles at collisions. The yield distribution of secondary electrons as a function of energies and angles of primary electron for the material coated on the inside wall of channel is also required.
Fig. 1. Structure of a tilted microchannel electron multiplier. Here $\theta$ is the tilt angle, $V_a$ is the applied voltage, $L$ is the length between faces of multiplier plate, and $D$ is the diameter of a channel.

Generally the secondary electron yield depends on the material, collision energy and incident angle at the surface. According to Ito et al. [9], [10], the yield has a Poisson distribution and its average value obeys the empirical formula

$$\delta_{\text{avg}} = \delta_{\text{max}} \cdot \left( \frac{4\pi}{x+1} \right) \exp(a(1 - \cos(\kappa)))$$

where

- $a$ material constant;
- $\kappa$ incident angle;
- $E_{\text{max}}$ energy for the maximum yield ($\delta_{\text{max}}$) at the normal incidence i.e., $\kappa = 0$.

For a lead glass, $a = 0.50$, $\delta_{\text{max}} = 4.0$, and $E_{\text{max}} = 250$ eV [9]. Consequently, the actual yield in simulations is obtained by a random sample from a Poisson distribution with the average value given by (1).

When secondary electrons are emitted from the wall they are independent of the history of electron trajectories [12], [13]. It is suggested that the emission angles of secondary electrons follow a cosine distribution to the normal direction of the surface and that the emission energies have the Maxwellian energy distribution [9], [10]. Thus, emission angles and energies will be sampled randomly from the given distributions by the MC method.

Electron trajectories can be simply calculated since electrons are assumed to move in a uniform electric field. The collision position, collision energy, and incident angle at the channel wall are determined as follows: The trajectories are calculated until the electrons pass the boundary (wall) of channel. The collision time is determined to within an allowed error through a tuning process using bisections. All components of velocity at the collision time give us the collision position, collision energy, and incident angle. This tuning method for collision time is useful for various-geometry channels.

III. SIMULATION RESULTS AND DISCUSSIONS

In simulations, we used a cylindrical channel with diameter $D = 100 \ \mu m$, length [the exact channel length = $L \cos(\theta)$] $L = 500 \ \mu m$, applied voltage $V_a = 1000 \ \text{V}$, and tilt angle $\theta = 0^\circ$ in general. These values were used default values in our simulations. We assumed that the secondary emissive material on the channel wall was lead glass and the average emissive energy was $6 \ \text{eV}$ [14]. When we measured the gain of the channel, we used 11 primary electrons injected with incident angles of $\kappa = 0$, $\pm5^\circ$, $\cdots$, $\pm25^\circ$, where $\kappa$ is an angle with respect to the channel axis $z$. The use of primary electrons with various angles reduces the possibility of a bias introduced by the angular dependence of gain. Primary electrons enter the channel with an initial energy of $100 \ \text{eV}$, which is based upon the field emitted electrons [15], at $100 \ \mu m$ away from the channel aperture. Statistics were taken for $1000 \ \text{MC}$ iterations for each data point and statistical errors, thereby, were less than the size of symbols in figures.

Fig. 2 shows trajectories of primary and secondary electrons for a nontilted ($\theta = 0^\circ$) cylindrical channel. Primary electrons were launched at $z = -100 \ \mu m$ with initial energy 100 eV. They enter the channel with incident angles of $\kappa = 0$, $\pm5^\circ$, $\cdots$, $\pm25^\circ$. Fig. 3 shows plots of gains of the nontilted cylindrical channel versus different applied voltages in (a) and different lengths in (b). The channel length was fixed at $500 \ \mu m$ in Fig. 3(a) while the applied voltage was fixed at $1000 \ \text{V}$ in Fig. 3(b). In Fig. 3(a) the gain is maximum near $V_a = 300 \ \text{V}$ but its variations are small. The relatively small variation in this case is due to the small aspect ratio of the channel, i.e., five. In Fig. 3(b) the gain is increased one thousand times near $L = 2000 \ \mu m$. For this short multiplier, it is summarized that the increase of the aspect ratio is more effective than the change of voltages for improving the gain.

Fig. 4 shows trajectories of primary and secondary electrons for a tilted cylindrical channel. The channel is tilted at $\theta = 10^\circ$ in $z$-direction with respect to the original channel axis $z$. Note that the electric field has still only $z$-component for the tilted channel, like the nontilted channel. Since the geometry of the tilted channels is not axis-symmetric, the primary electrons are injected with the various incident angles $\kappa$ and an angle $\phi = 45^\circ$, where $\phi$ is an azimuthal angle with respect to the $x$-axis on the transverse plane, $xy$, of the channel. This angle $\phi$ affects the initial interception on the wall for primary electrons. Usually the gain for a primary electron with $\phi = 45^\circ$ incident azimuthal
Fig. 3. (a) Plot of the gain as a function of applied voltages for a nontilted cylindrical channel at a fixed length $L = 500 \, \mu m$. (b) A plot of the gain as a function of channel lengths for a cylindrical channel at a fixed applied voltage $V_a = 1000 \, V$.

Fig. 4. Electron trajectories for a tilted channel ($\theta = 10^\circ$). Primary electrons start at $z = -100 \, \mu m$ with initial energy 100 eV. They enter the channel with incident angle from $\kappa = 0, \pm 5, \pm 25^\circ$ and $\phi = 45^\circ$. Here, $\kappa$ is the angle with respect to the $z$-axis and $\phi$ is an azimuthal angle on the transverse plane, $xy$-plane.

angle is between values for $\phi = 90^\circ$ and $\phi = 0^\circ$. The trajectories of most electrons for the tilted channel in Fig. 4 are biased to one side of the channel wall while symmetric for the nontilted channel in Fig. 2.

Fig. 5(a) shows plots of gains versus various tilt angles ($\theta$) for the tilted channel with different applied voltages $V_a = 500, 1000$ and $1500 \, V$. Gains are improved as the tilt angle is increased until it reaches a characteristic angle. The gain has a peak near $\theta = 25^\circ$ and the maximum values are up to $10^4$ times larger than those of the nontilted case are ($\theta = 0^\circ$).

For tilted channels the actual channel length is $L \cos(\theta)$ but the effect of the change of the channel length on the gain is not significant for $\theta < 30^\circ$. For $\theta > 35^\circ$ no gain appears. Gains of tilted channels for different lengths, $L = 500, 1000$ and $1500 \, \mu m$, in Fig. 5(b) are improved, too. As the applied voltage increases, so do gains before the peak point, around $\theta = 25^\circ$. This is, however, not true after the peak. The tilt of the channel with respect to the electric field improved the gain significantly. This can be explained as follows.

For the nontilted channel the electric field has only the axial component, provided that the fringe field is neglected near the input and output aperture of the channel. Consequently, there is a uniform field in the axial direction. When a primary electron enters the channel with some incident angle, it strikes a point of the channel wall and induces the emission of secondary electrons. Emitted secondary electrons generally hit the opposite side of the wall. Fig. 6(a) shows electron trajectories in a straight channel when secondary electrons are assumed to emit only at the normal direction to the channel wall. The trajectories are collectively on a zigzag mode. For a short channel the number of hits of the wall is small and gains are not high. This is due to the fact that the gain usually depends on the channel length and applied voltage.

When a channel is tilted, the only change is the angle between the channel axis and the electric field. The change in gain is, however, outstanding. Fig. 6(b) illustrates trajectories of electrons emitted at the normal direction of a tilted channel wall. The motion of electrons is collectively on a hopping mode along one side of the channel wall. The mode may be due to an effective field on the channel wall. The electric field ($E$) due to applied voltage can be separated into the tangential component ($E_t$) and...
When channels are tilted to the channel axis, the collective motion of electrons in a tilted channel is in a hopping mode while usually in a zigzag mode for conventional straight channels. This different mode is effective on the product of high gains in a short channel. This microchannel electron multiplier with tilted structures will be useful for applying the electron multiplier to various practical areas.

**References**


